Worksheet 3:

The amplitude of the gravitational waves

One property of gravitational waves is that they cyclically stretch and compress objects along an axis perpendicular to their direction of propagation. The change in length induced by a gravitational wave is precisely what the huge laser interferometers are looking for. Although the quantity of energy emitted in the form of gravitational waves is enormous in the immediate vicinity of the pair of orbiting black holes, only a minuscule fraction of this energy is directed towards the Earth.

Your next task is to calculate the order of magnitude of the expected change in length, or in other words the amplitude of the gravitational wave event GW140915. Unfortunately, classical physics cannot help us with this, so you will need to use a formula from general relativity. Assume that both masses are of equal size and that they move around one another in circles.

With these assumptions, the amplitude h (strain) is given by $h = \frac{5.255 \cdot 10^{-17} \cdot (f_{GW})^{\frac{2}{3}} \cdot (M)^{\frac{5}{3}}}{\text{Distance (ly)}}$.

The frequency f_{GW} of the gravitational waves is expressed in Hertz, the total mass M is expressed in solar masses, and the distance from Earth is expressed in light years (ly). Since the amplitude is viewed as a relative change in length $\Delta L / L$, the parameter h is a dimensionless quantity.

Consider the time interval between 0.35 and 0.4 seconds in the diagram. The average frequency of the gravitational wave on this interval is approximately 50 Hertz. The total mass of the system is assumed to be M = 65 solar masses, and the system is assumed to be 1.3 billion light years away from Earth.

a) Using these values, find the amplitude of the gravitational wave when it reaches Earth.

b) Compare your result with the values recorded by the detectors of the laser interferometer. You should find a value that is slightly greater than $h = 0.5 \cdot 10^{-21}$.

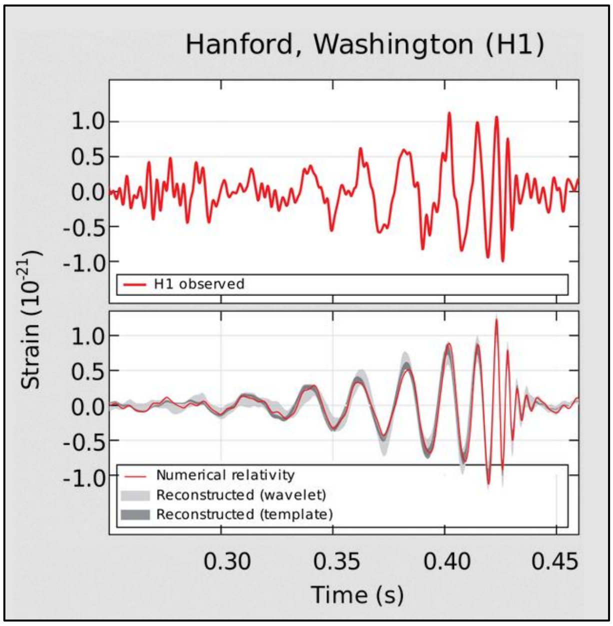

Image source: https://commons.wikimedia.org/wiki/File:LIGO_measurement_of_gravitational_waves.svg

c) The amplitude h describes the relative change in length of an object traversed by the gravitational wave.

This amplitude satisfies the relation: $h = \frac{\Delta L}{L}$.

The arms of the interferometers in Hanford and Livingston are 4,000 metres long. Find the change in the length ΔL, and compare your value to the diameter of a proton ($d_{Proton} \approx 0.9 \cdot 10^{-15}\,m$).

This shows just how difficult it is for scientists to detect gravitational waves!