Worksheet 4:

Where was the source?

The binary orbits and subsequent merging of the two black holes 1.3 billion light years away were completely invisible from Earth on the electromagnetic spectrum. Nevertheless, scientists were able to roughly identify the location of the source, since the signal was recorded by two interferometers that were sufficiently distant from one another.

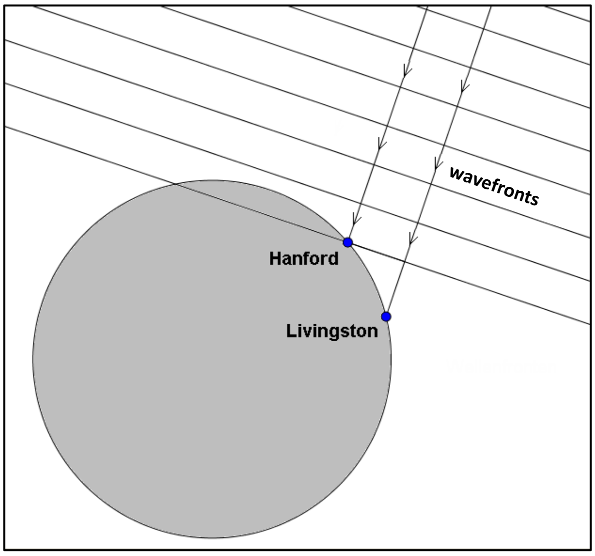


Image source: M. Borchardt

The figure shows how the wavefronts of the gravitational waves struck the Earth. The waves hit the interferometer in Hanford first, then registered on the detector in Livingston (in the south of the USA) 3,000 km away, after a delay. This time delay can be used to deduce the angle from which the waves hit the Earth.

a) The graphs of the gravitational waves show that the signal arrived later in Livingston than in Hanford. Estimate the duration of the time delay Δt .

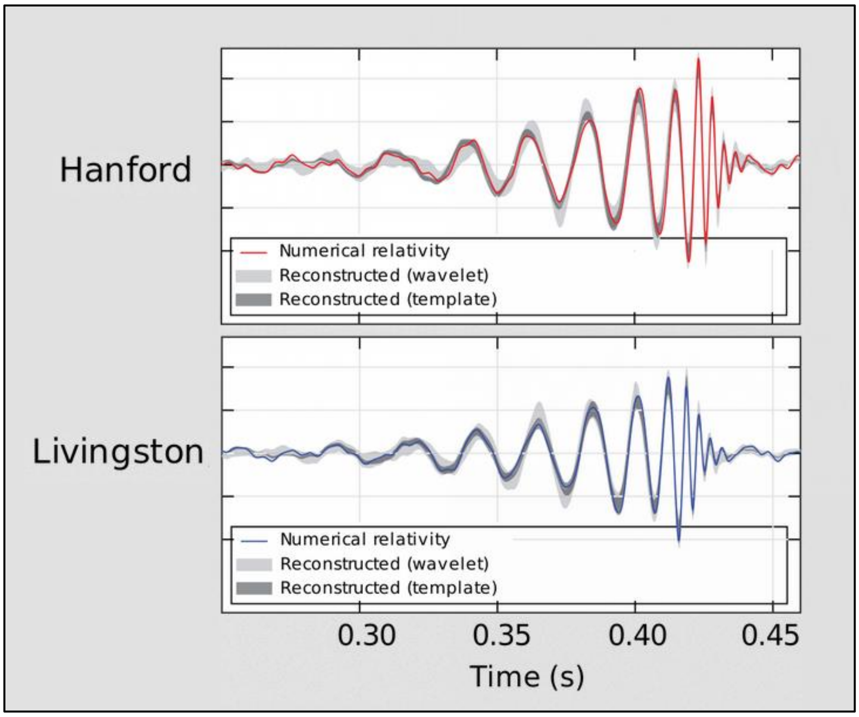


Image source: https://commons.wikimedia.org/wiki/File:LIGO_measurement_of_gravitational_waves.svg

b) A precise analysis found a value of $\Delta t = 6.9\,\mathrm{ms}$. Calculate the length difference Δs in the paths travelled by the wavefronts, assuming that they propagate at the speed of light.

c) The distance D separating the two interferometers is almost exactly 3,000 km. Use the figure to deduce the angle α.

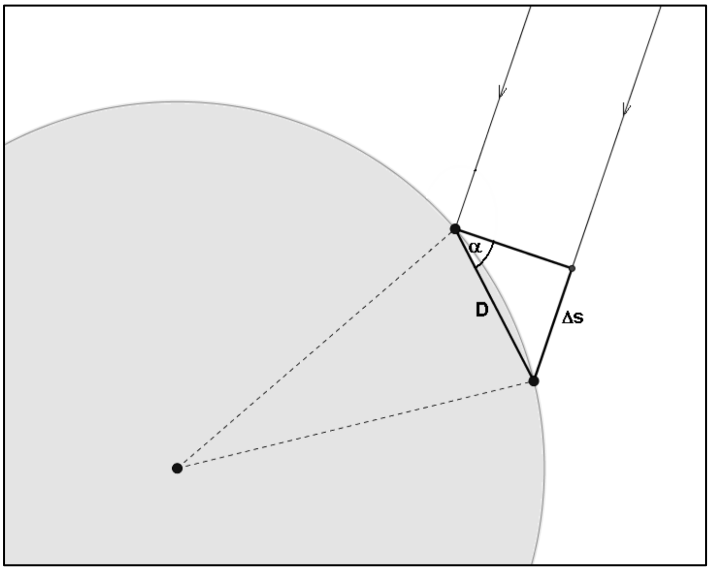


Image source: M. Borchardt

Remark:

As you can see in the diagram below, simply knowing the angle α is not enough to fully specify the position of the source. Finding this angle only restricts the possible origins of the signal to a circular or ribbon-shaped region in the sky; a third detector is required to identify the location of the source more precisely. As it happens, a third large interferometer has since been opened at the VIRGO facility in Italy. The readings from this facility helped to narrow down the more recently discovered gravitational source GW170814 to an extremely precise region of space.

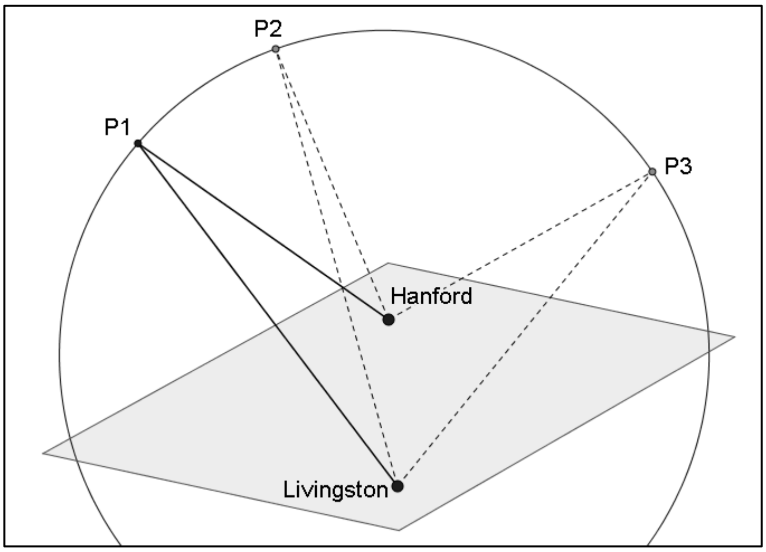
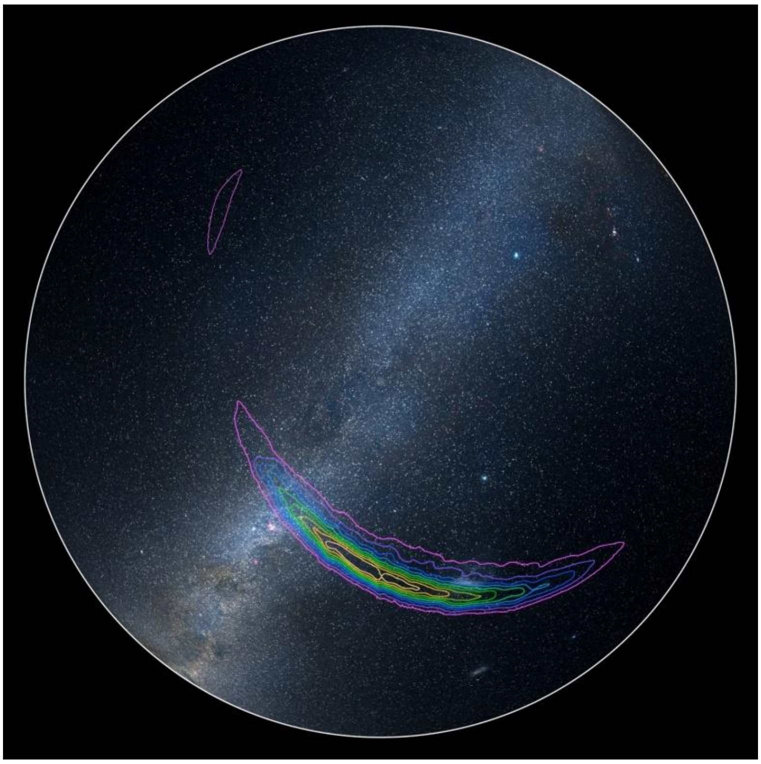


Image source: M. Borchardt

At the time of the first gravitational wave event in 2015, the source could only be located with limited accuracy, as you can see in the figure below.



Location of the source in the sky
Image source: https://www.ligo.caltech.edu/image/ligo20160211b

Other interesting notes

The sensitivity of interferometric gravitational wave detectors depends on:

1. the direction from which the gravitational waves hit the detector;
2. the orientation of the detectors relative to the polarization of the gravitational wave.

Remarks for 1.:

Gravitational waves arriving from above or below the plane of the detector arms are the easiest to "see", since the relative compression and stretching of the arms is maximized. The interferometer is significantly less sensitive to waves arriving from the side, i.e. waves whose direction of propagation lies in the plane of the detector.

The interferometer also has four blind spots: waves arriving from the directions along the bisectors of the L-shaped array are completely invisible, since they stretch and compress both detector arms by an equal amount at any given moment in time, meaning that the light waves are not shifted relative to one another.[1]

(Incidentally, the gravitational waves in the video presentation appear to arrive at the interferometer from one of its bisectors, which would be unfavourable for detection.)

Remarks for 2.:

Gravitational waves are polarized; in other words, they stretch and compress space-time in two perpendicular directions. These directions remain constant over time. If the wave arrives from above (or below) the plane of the arms, and the direction of the amplitude of the waves is perfectly aligned with the bisector of the two detector arms, then the arms will be compressed and stretched by equal amounts, once again nullifying any shift in the light waves.

The third large laser interferometer VIRGO in northern Italy was deliberately oriented at an angle of approximately 45° relative to the two LIGO facilities. Together, all three systems should therefore be able to provide us with information about the direction of polarization of gravitational waves in the future.

[1] https://www.ligo.org/science/Publication-GW170814/translations/science-summary-german.pdf